CPT Symmetry and
The Equality of Mass and Lifetime

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Introduction

CPT Theorem

- **CPT Theorem: imply mass and lifetime equality**

- **But,**

  Is not mass and lifetime equality required before for CPT symmetry to be valid?

Symmetry

- Charge conjugate(C) symmetry:

  Particle and antiparticle equations are the same under the transformation of fields.

  Ex)
$$\begin{cases} \psi \longrightarrow \psi^{c} \\ A_{\mu} \longrightarrow A_{\mu}^{c} \end{cases}$$

- Particle and antiparticle symmetry:

  Mass and lifetime of particle and antiparticle are the same.

Symmetry

- **Particle and Antiparticle: by the Dirac Eq.**

$$(i\partial\!\!\!/ - m)\psi = +e A_\mu \gamma^\mu \psi \qquad \text{particle}$$

$$(i\partial\!\!\!/ - \overline{m})\psi^c = -\overline{e} A^c_\mu \gamma^\mu \psi^c \qquad \text{antiparticle}$$

- Mass and coupling constants of particle and antiparticle: expected to be the same, but could be independent

Symmetry

- Particle: under the C transformation of ψ and A

$$\begin{cases} \psi \longrightarrow \psi^{c} \\ A_\mu \longrightarrow A_\mu^{c} \end{cases}$$ on particle Eq.

$$(i\partial - m)\psi^{c} = -e A_\mu^{c}\gamma^{\mu}\psi^{c}$$ Transformed particle

$$(i\not\partial - \overline{m})\psi^{c} = -\overline{e} A_\mu^{c}\gamma^{\mu}\psi^{c}$$ Antiparticle

- C symmetry requires:

$$\begin{cases} m = \overline{m} \\ e = \overline{e} \end{cases}$$

C Violation

- Type 1: if $m \neq \bar{m}$ or $e \neq \bar{e}$

  → Particle and antiparticle symmetry violation

- Regardless of C transformation on ψ and A,

  if $m \neq \bar{m}$ or $e \neq \bar{e}$, then Eqs. can NOT be the same.

$$(i\partial\!\!\!/ - m)\psi = +e A_\mu \gamma^\mu \psi \qquad \text{particle}$$

$$(i\partial\!\!\!/ - \bar{m})\psi^c = -\bar{e} A_\mu^c \gamma^\mu \psi^c \qquad \text{antiparticle}$$

C Violation

- Type 2: Different signs for coupling constants

$$(i\partial\!\!\!/ - m)\psi = +eA_\mu\gamma^\mu\psi + g_1 A_\mu\Gamma^\mu\psi + g_2 B_\mu\Gamma^\mu\psi$$

$$(i\partial\!\!\!/ - m)\psi^c = -eA_\mu^c\gamma^\mu\psi^c + g_1 A_\mu^c\Gamma^\mu\psi^c + g_2 B_\mu^c\Gamma^\mu\psi^c$$

different after C transformation

- Particle and antiparticle symmetry

$$\tau \propto |\mathcal{M}_e|^2 + |\mathcal{M}_{g_1}|^2 + |\mathcal{M}_{g_2}|^2$$

or

$$\tau \propto |\mathcal{M}_e \pm \mathcal{M}_{g_1}|^2 + |\mathcal{M}_{g_2}|^2$$ interference

Different lifetime for particle and antiparticle

C Violation

- C Violation but Mass and lifetime Equality

  Ex) Type 2 C violation without interference

- Particle and antiparticle violation with C symmetry

  Ex) C symmetry but due to phase in the interference term

$$\tau \propto |\mathcal{M}_e \pm \mathcal{M}_{g_1}|^2 + |\mathcal{M}_{g_2}|^2$$

interference

Irrelevant to the relative signs between e , g_1

C Violation

- Why C symmetry is irrelevant of lifetime?

Answer:
In defining C symmetry, we consider

$$(i\not{\partial} - m)\psi = \boxed{\pm e A_\mu \gamma^\mu \psi}$$ part of equations

But, in the lifetime calculation,

$$(i\not{\partial} - m)\psi = 0 \quad \text{and} \quad \pm e A_\mu \gamma^\mu \psi$$

The lifetime is independent of the signs of interactions

Proof of Mass Equality

□ Theorem:

Mass equality between particle and antiparticle

Proof by T.D. Lee:

m: z comp. angular momentum

i) $$mass_p = \langle p \,|\, H \,|\, p \rangle_m$$

$$= \langle p \,|\, H \,|\, p \rangle_m^*$$

H: real, independent of m

$$= \langle p \,| \Theta^{-1} \Theta H \Theta^{-1} \Theta | p \rangle_m$$

$$\Theta | p \rangle_m = e^{i\theta} | \overline{p} \rangle_m$$

$$\Theta H \Theta^{-1} = H$$ CPT Symmetry

$$= \langle \overline{p} \,|\, H \,|\, \overline{p} \rangle_{-m}$$

Proof of Mass Equality

- Theorem:

Mass equality between particle and antiparticle

Proof by T.D. Lee :

ii) $$mass_p = \langle p \,|\, H \,|\, p \rangle_m$$

$$= \langle \bar{p} \,|\, H \,|\, \bar{p} \rangle_{-m}$$

$$mass_{\bar{p}} = \langle \bar{p} \,|\, \bar{H} \,|\, \bar{p} \rangle_{-m}$$

Therefore,

$$mass_p = mass_{\bar{p}}$$

when $H = \bar{H}$

Proof of Mass Equality

- The flaw in Mass Equality proof by T.D. Lee:

i) The proof is based on C symmetry, not CPT symmetry as claimed

Ex) Masses can be different with CPT symmetry if C violation.

If $$CHC^{-1} = -H \neq H$$

→ $$mass_{p} \neq mass_{\bar{p}}$$

Proof of Mass Equality

- The flaw in Mass Equality proof by T.D. Lee:

ii) To evaluate equality between particle and antiparticle:

$$(i\not{\partial} - m)\psi = 0$$

$$(i\not{\partial} - m)\psi^{CPT} = 0$$

$$(i\partial - m)\psi^{C} = 0$$

Eqs. to compare

Not necessary unless

$$\psi^{CPT} = \psi^{C}$$

which is only an assumption

Proof of Mass Equality

- The flaw in Mass Equality proof by T.D. Lee:

iii) Mass equality can be proved only by C symmetry

If $H = \overline{H} = C H C^{-1}$

$$mass_p = \langle p \mid H \mid p \rangle_m$$

$$= \langle p \mid C^{-1} C H C^{-1} C \mid p \rangle_m$$

$$= \langle \overline{p} \mid \overline{H} \mid \overline{p} \rangle_m$$

$$= mass_{\overline{p}}$$

Proof of Mass Equality

- The flaw in Mass Equality proof by T.D. Lee:

iv) Mass equality is prerequisite for C symmetry, not a consequence that can be proved by C or CPT

To be $\mathcal{L}_0 \rightarrow \overline{\mathcal{L}}_0$,

$$\mathcal{L}_0 = \overline{\psi}\,(i\partial\!\!\!/ - m)\,\psi$$

$$\overline{\mathcal{L}}_0 = \overline{\psi}\,(i\partial\!\!\!/ - \overline{m})\,\psi$$

we need $m = \overline{m}$ first

Proof of Lifetime Equality

- Consider

$$H = \underline{H_{strong}} + H_{weak}$$

C,P,T invariant

- Lee & Yang Theorem

If a particle A decay through the interaction H_{weak} , and if the lifetimes of A and antiparticle A are the same, even if H_{weak} is not invariant under charge conjugate.

Proof of Lifetime Equality

- Lee & Yang Theorem

Proof:

$$\langle B | H_{weak} | A \rangle^* = \langle TB | T H_{weak} T^{-1} | TA \rangle$$

CPT

$$= \langle B | C^{-1} P^{-1} H_w P C | A \rangle$$

$$= \pm \langle \overline{B} | H_{weak} | \overline{A} \rangle$$

Therefore, the lifetimes of A and $\overline{A}$ are equal

Proof of Lifetime Equality

- The lifetime equality with CPT Violation:

  using Lee & Yang proof

Proof:

CPT Violation

$$\langle B | H_{weak} | A \rangle^{*} = \langle TB | TH_{weak} T^{-1} | TA \rangle$$

$$= -\langle B | C^{-1} P^{-1} H_{w} P C | A \rangle$$

$$= \mp \langle \overline{B} | H_{weak} | \overline{A} \rangle$$

Therefore, the lifetimes of A and $\overline{\text{A}}$ are equal
with CPT violation

Proof of Lifetime Equality

- The flaw of lifetime equality Theorem

No consideration for C violation Type 1!

i) If $m \neq \overline{m}$ or $e \neq \overline{e}$,

NO C transformation on the field of H gets to $\overline{H}$

$$\overline{H} \neq C H C^{-1}$$

thus $|\overline{A}\rangle \neq C|A\rangle$

ii) If $e \neq \overline{e}$, simply

$$\tau \neq \overline{\tau}$$

Proof of Lifetime Equality

- Coupling constants equality

→ The lifetime equality regardless of symmetry!

$$\tau \propto |\mathcal{M}_W|^2 + |\mathcal{M}_g|^2$$

No interference assumed

→ Why do we care the sign in $|\ \ |^2$?

τ is the same regardless of this sign!

- This is due to our practical calculation

$$(i\partial\!\!\!/ - m)\psi = 0 \quad \text{and} \quad \pm e A_\mu \gamma^\mu \psi$$

→ Free particle, No info. on its charge!

Summary

- C Symmetry, Particle and antiparticle symmetry
- Different types of C violation
- Mass equality: C symmetry, not CPT
- Lifetime equality: regardless of C, CPT violation
- Comparison of mass between particle and antiparticle:

  Paricle and antiparticle symmetry test,
  not CPT Test!